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Effects of Confinement on Interaction Diagrams of Square Reinforced Concrete Columns

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Abstract: To prevent brittle failure, the design of a structural column in a seismic-resistant building is of important consideration, particularly in terms of confinement. In the recent building code, the need of closely-spaced stirrups in a structural member, such as column becomes compulsory due to the ductility and strength considerations. However, the design is based on the simplified block stress of unconfined concrete, and does not account for the strength gain due to the presence of confinement. To investigate the effects of lateral confinement on the column capacity, an analytical study is carried out. Both the strength gain in concrete core and the loss of strength in the cover are considered in the analytical models to exhibit the remaining strength gain after the mobilization of strength gain in the core concrete to compensate the loss of strength in the concrete cover. There are six key parameters primarily influence the effectiveness of lateral confinement. The most influencing parameter is found to be the spacing of transverse steel. The presence of closely-spaced lateral confinement significantly increases the magnitude of stress-strain curve of concrete. This increase expands the interaction diagram of the column particularly when it is in the compression-controlled region (for lower-story columns when axial load dominates the behavior).

Keywords: column capacity, confinement effects, interaction diagram, lateral reinforcement..

Introduction

The effects of confinement on a structural column in a building are mainly due to the presence of lateral reinforcement provided over the column height. It results in higher capacity and ductility of a column that help to prevent the column from brittle failure. Laterally-confined columns have higher capacity than the unconfined ones since the concrete core of the columns gains the strength from the mobilization of lateral confinement. Recent development in research and design engineering, particularly in reinforced concrete structures often requires higher capacity and ductility of structural members. To prevent a building structure from brittle failure, the design of a structural column in a seismic-resistant building is of important consideration, particularly in terms of confinement [1].

Up to present, the design of a structural column is based on the simplified block stress of unconfined concrete proposed by Whitney [2]. This proposed block stress was adopted by the ACI Building Code since 1956 edition [3], and it has been in the code since then. The concept was also adopted in the national building code [4] for flexural design. In SNI 03-2847-2002 [5], it remains applicable for flexural design of reinforced concrete members. The existing interaction diagrams developed for the column capacity are also based on this assumption that does not account for the strength gain from the presence of confinement. Even though the block stress concept has long been adopted as a reasonable approach, the research indicated that the presence of confinement in a concrete column would affect the actual compressive stress-strain curve of concrete. This effort gives a more accurate prediction on the compressive force of concrete in a column, and thus, resulting further in a more efficient column cross-section [6]. With advancement of computer programming and technology, the computational effort can be much accelerated by implementing the numerical procedure to solve the stress-strain curves.

To investigate the effects of lateral confinement on the column capacity, an analytical study is carried out. Both the strength gain in concrete core and the loss of strength in the cover are considered in the analytical models to exhibit the remaining strength. So far, this strength gain in the confined core is used only for the compensation of the possible strength loss due to the spalling of concrete cover (which is unconfined). Recent codes of practice still disregard this effect for the design purposes and, thus in the

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conservative side. In this numerical study of confined concrete columns, the behavior of concrete core is modeled by the stress-strain relationship of confined concrete, whereas for the cover, as unconfined concrete. Several stress-strain relationships of confined concrete available in literature are adopted in the study, namely Kent-Park [7], Sheikh-Uzumeri [8], Mander et al. [9], Yong-Nawy [10], Cusson-Paultre [11], Diniz-Frangopol [12], Kappos-Konstantinidis [13], Hong-Han [14], and Kusuma-Tavio [15].

From the study, it can be concluded that there are six key parameters primarily influence the effectiveness of lateral confinement. The most influencing parameter is found to be the spacing of transverse steel. Even though, the codes ignore the effect of confinement on the strength gain due to the conservative consideration for the design purposes, the authors still intend to discover the actual possible remaining gain of strength due to the presence of confinement.

Unconfined Concrete Models

The unconfined concrete models adopted in the study are Whitney's block stress [2], Kent-Park [7], Popovics [16], and Thorensfeldt et al. [17] models. Brief summary of these models are described subsequently.

Whitney's Block Stress [2]

This model is only used at the ultimate limit state. The compressive stress of concrete is asummed to be constant as a block stress at the following value:

$$f_c = 0.85\, f_c' \tag{1}$$

$$a = \beta_1 c \tag{2}$$

where:

$$\beta_1 = \begin{cases} 0.85 & \text{if } f_c' \le 30 \text{ MPa} \\ 0.85 - (f_c' - 30)\dfrac{0.05}{7} & \text{if } 30 \text{ MPa} < f_c' \le 58 \text{ MPa} \\ 0.65 & \text{if } f_c' > 58 \text{ MPa} \end{cases}$$

$$\varepsilon_{cu} = 0.003$$

in which, c is the distance of neutral axis from extreme compressive fiber of concrete, β_1 the conversion factor from parabolic to rectangular shape as a function of concrete compressive strength, and ε_{cu} the ultimate strain of unconfined concrete.

Kent-Park Model [7]

For ascending branch, $\varepsilon_c \le \varepsilon_{co}$ ($\varepsilon_{co} = 0.002$):

$$f_c = f_c'\left[\frac{2\varepsilon_c}{\varepsilon_{co}} - \left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right] \tag{3}$$

For descending branch, $\varepsilon_c \ge \varepsilon_{co}$:

$$f_c = f_c'\left[1 - Z_0(\varepsilon_c - \varepsilon_{co})\right]$$

where:

$$Z_0 = \frac{0.5}{\varepsilon_{50u} - \varepsilon_{co}}$$

$$\varepsilon_{50u} = \frac{3 + 0.002\, f_c'}{f_c' - 1000}$$

in which, ε_{50u} is the unconfined concrete strain when the stress reaches 50 percent of peak stress, ε_{co} the unconfined concrete strain at the peak stress, and f_c' the compressive strength of unconfined concrete (in psi, 1 psi = 0.006895 MPa).

Popovics Model [16]

For entire stress-strain curve of concrete, the stress is calculated using the following equation:

$$f_c = f_c'\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)\frac{n}{\left[n - 1 + \left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^n\right]} \tag{5}$$

where:

$$\varepsilon_{co} = 0.0005(f_c')^{0.4}$$

$$n = 0.8 + \frac{f_c'}{17}$$

All units are in continental system, where 1 psi = 0.006895 MPa.

Thorenfeldt et al. Model [17]

For entire stress-strain curve of concrete, the stress is calculated using the following equation:

$$f_c = f_c'\left(\frac{\varepsilon_c}{\varepsilon_{co}}\right)\frac{n}{\left[n - 1 + \left(\dfrac{\varepsilon_c}{\varepsilon_{co}}\right)^{nk}\right]} \tag{6}$$

where:

$$k = \begin{cases} 1 & \text{if } \dfrac{\varepsilon_c}{\varepsilon_{co}} \le 1 \\ 0.67 + \dfrac{f_c'}{62} & \text{if } \dfrac{\varepsilon_c}{\varepsilon_{co}} > 1 \end{cases}$$

$$n = 0.8 + \frac{f_c'}{17}$$

$$E_c = 3320\sqrt{f_{co}'} + 6900 \text{ (in MPa)}$$

$$\varepsilon_{co} = \frac{f_c'}{E_c}\left(\frac{n}{n-1}\right)$$

Confined Concrete Models

The confined concrete models adopted in the study are Kent-Park [7], Sheikh-Uzumeri [8], Mander et al. [9], Yong-Nawy [10], Cusson-Paultre [11], Diniz-Frangopol [12], Kappos-Konstantinidis [13], Hong-Han [14], and Kusuma-Tavio [15] models. The most obvious difference of all these confined stress-strain models is particularly in term of ductility along the descending branch [18]. Brief summary of these models are described subsequently.

Kent-Park Model [7]

For ascending branch, $\varepsilon_c \le 0.002$:

$$f_c = f_c'\left[\frac{2\varepsilon_c}{0.002} - \left(\frac{\varepsilon_c}{0.002}\right)^2\right] \quad (7)$$

For descending branch, $0.002 \le \varepsilon_c \le \varepsilon_{20c}$:

$$f_c = f_c'\left[1 - Z\left(\varepsilon_c - 0.002\right)\right] \quad (8)$$

For horizontal branch, $\varepsilon_c \ge \varepsilon_{20c}$:

$$f_c = 0.2\, f_c' \quad (9)$$

where:

$$Z = \frac{0.5}{\varepsilon_{50u} + \varepsilon_{50h} - 0.002}$$

$$\varepsilon_{50u} = \frac{3 + 0.002\, f_c'}{f_c' - 1000}$$

$$\varepsilon_{50h} = \frac{3}{4}\rho_s\sqrt{\frac{b''}{s_h}}$$

where ρ_s is the volumetric ratio of lateral reinforcement to the confined concrete core measured outer-to-outer of lateral reinforcement, b'' the width of confined concrete core measured outer-to-outer of lateral reinforcement, and s_h the spacing of lateral reinforcement.

Sheikh-Uzumeri Model [8]

For ascending branch, $\varepsilon_c \le \varepsilon_{cc1}$:

$$f_c = f_{cc}'\left[2\left(\frac{\varepsilon_c}{\varepsilon_{cc}}\right) - \left(\frac{\varepsilon_c}{\varepsilon_{cc}}\right)^2\right] \quad (10)$$

For horizontal branch, $\varepsilon_{cc1} < \varepsilon_c \le \varepsilon_{cc2}$:

$$f_c = K_s\, f_{co}' \quad (11)$$

For descending branch, $\varepsilon_{cc2} < \varepsilon_c \le \varepsilon_{cc30}$:

$$f_c = f_{cc}'\left[1 - Z\left(\varepsilon_c - \varepsilon_{cc}\right)\right] \quad (12)$$

where:

$$Z = \frac{0.5}{\frac{3}{4}\rho_s\sqrt{\frac{b_c}{s}}}$$

$$f_{cc}' = K_s\, f_{co}'$$

$$f_{co}' = 0.85\, f_c'$$

$$\varepsilon_{cc1} = 80 K_s\, f_c' \times 10^{-6}$$

$$\varepsilon_{cc2} = \varepsilon_{co}\left[1 + \frac{248}{C}\left[1 - 5.0\left(\frac{s}{b_c}\right)^2\right]\frac{\rho_s f_s'}{\sqrt{f_c'}}\right]$$

$$K_s = 1.0 + \frac{(b_c)^2}{140 P_{occ}}\left[\left(1 - \frac{nC^2}{5.5 b_c^2}\right)\left(1 - \frac{s}{2b_c}\right)^2\right]\sqrt{\rho_s f_s'}$$

$$P_{occ} = f_{co}'\left(A_{cc}\right)$$

$$\varepsilon_{co} = 0.0022$$

$$\varepsilon_{cc85} = 0.225\rho_s\sqrt{\frac{b_c}{s}} + \varepsilon_{cc2}$$

in which, A_{cc} is the area of confined concrete core, b_c the width of confined concrete core measured center-to-center of lateral reinforcement, C the distance between longitudinal reinforcement confined laterally by lateral reinforcement, K_s the magnification factor, f_s' the stress in lateral reinforcement at the maximum stress of confined concrete (assume $f_s' = f_{yh}$ at the peak stress), and n the number of ineffective parabolic area in concrete core, or the number of longitudinal reinforcement confined laterally by lateral reinforcement.

Mander et al. Model [9]

For entire stress-strain curve of concrete, the stress is calculated using the following equation:

$$f_c = \frac{f_{cc}' x r}{r - 1 + x^r} \quad (13)$$

where:

$$x = \frac{\varepsilon_c}{\varepsilon_{cc}}$$

$$r = \frac{E_c}{E_c - E_{\sec}}$$

$$E_c = 5000\sqrt{f_c'} \text{ MPa}$$

$$E_{\sec} = \frac{f_{cc}'}{\varepsilon_{cc}}$$

$$\varepsilon_{cc} = \varepsilon_{co}\left[1 + 5\left(\frac{f_{cc}'}{f_c'} - 1\right)\right]$$

$$\varepsilon_{co} = 0.002$$

$$f'_{cc} = f'_c\left(-1.254+2.254\sqrt{1+\frac{7.94f'_\ell}{f'_c}}-2\frac{f'_\ell}{f'_c}\right)$$

$$K_e = \frac{A_e}{A_{cc}}$$

$$f'_{\ell x} = k_e\rho_x f_{yh} \text{ (}x\text{-direction)}$$

$$f'_{\ell y} = k_e\rho_y f_{yh} \text{ (}y\text{-direction)}$$

$$K_e = \frac{\left(1-\sum_{i=1}^{n}\frac{(w'_i)^2}{6b_cd_c}\right)\left(1-\frac{s'}{2b_c}\right)\left(1-\frac{s'}{2d_c}\right)}{(1-\rho_{cc})}$$

$$\varepsilon_{cu} = 0.004+1.4\rho_s f_{yh}\varepsilon_{sm}/f'_{cc}$$

in which, b_c, d_c is the cross-sectional dimension of confined concrete core measured center-to-center of lateral reinforcement in the x and y directions, respectively, s' the clear spacing of lateral reinforcement, A_e the effective area of confined concrete core, w'_i the i^{th} clear spacing from two adjacent longitudinal reinforcement, ρ_{cc} the ratio of cross-sectional area of longitudinal reinforcement to area of confined concrete core, and ε_{sm} the strain of reinforcing steel at maximum tensile stress.

Yong et al. Model [10]

For ascending branch, $\varepsilon_c \le \varepsilon_{cc}$:

$$Y = \frac{AX+BX^2}{1+(A-2)X+(B+1)X^2} \quad (14)$$

For descending branch, $\varepsilon_c \ge \varepsilon_{cc}$:

$$Y = \frac{CX+DX^2}{1+(C-2)X+(D+1)X^2} \quad (15)$$

where:

$$X = \frac{\varepsilon_c}{\varepsilon_{cc}}$$

$$Y = \frac{f_c}{f'_{cc}}$$

$$A = E_c\frac{\varepsilon_{cc}}{f'_{cc}}$$

$$B = \left[\frac{(A-1)^2}{0.55}\right]-1$$

$$E_c = 36.78w_c^{1.5}\sqrt{f'_c}$$

$$C = \frac{(\varepsilon_{2i}-\varepsilon_i)}{\varepsilon_{cc}}\left[\frac{\varepsilon_{2i}E_i}{(f'_{cc}-f_i)}-\frac{4\varepsilon_iE_{2i}}{(f'_{cc}-f_{2i})}\right]$$

$$D = (\varepsilon_i-\varepsilon_{2i})\left[\frac{E_i}{(f'_{cc}-f_i)}-\frac{4E_{2i}}{(f'_{cc}-f_{2i})}\right]$$

$$E_i = \frac{f_i}{\varepsilon_i}$$

$$E_{2i} = \frac{f_{2i}}{\varepsilon_{2i}}$$

$$f'_{cc} = K_s f'_c = \left[\begin{matrix}1+0.11\left(1-\frac{0.254s}{h''}\right)\\ \left(\rho_s+\frac{n\phi_s}{0.31496s\phi_\ell}\rho\right)\frac{f_{yh}}{\sqrt{f'_c}}\end{matrix}\right]f'_c$$

$$\varepsilon_{cc} = 0.00265+\frac{0.0035\left(1-\frac{0.734s}{h''}\right)(145\rho_s f_{yh})^{2/3}}{\sqrt{145f'_c}}$$

$$f_i = f'_{cc}\left[0.25\left(\frac{f'_c}{f'_{cc}}\right)+0.4\right]$$

$$\varepsilon_i = K_s\left[1.4\left(\frac{\varepsilon_{cc}}{K_s}\right)+0.0003\right]$$

$$f_{2i} = f'_{cc}\left[0.025\left(\frac{f'_c}{1000}\right)-0.065\right] \ge 0.3f'_{cc}$$

$$\varepsilon_{2i} = 2\varepsilon_i-\varepsilon_{cc}$$

in which, h'' is the width of confined concrete core measured inner-to-inner of lateral reinforcement, n the number of longitudinal reinforcement, ϕ_s the nominal diameter of lateral reinforcement, ϕ_ℓ the nominal diameter of longitudinal reinforcement, ρ the ratio of cross-sectional area of longitudinal reinforcement to gross area of concrete (A_s/A_g), and w_c the concrete density in kN/m^3. All units are in continental system (1 psi = 0.006895 MPa).

Cusson-Paultre Model [11]

For ascending branch, $\varepsilon_c \le \varepsilon_{cc}$:

$$f_c = f'_{cc}\left[\frac{k(\varepsilon_c/\varepsilon_{cc})}{k-1+(\varepsilon_c/\varepsilon_{cc})^k}\right] \quad (16)$$

For descending branch, $\varepsilon_c \ge \varepsilon_{cc}$:

$$f_c = f'_{cc}\exp\left[k_1(\varepsilon_c-\varepsilon_{cc})^{k_2}\right] \quad (17)$$

where:

$$k = \frac{E_c}{E_c - (f'_{cc}/\varepsilon_{cc})}$$

$$E_c = 3320\sqrt{f'_c} + 6900$$

$$k_1 = \frac{\ln 0.5}{(\varepsilon_{cc50} - \varepsilon_{cc})^{k_2}}$$

$$k_2 = 0.58 + 16\left(\frac{f_{\ell e}}{f'_{co}}\right)^{1.4}$$

$$\frac{f'_{cc}}{f'_{co}} = 1.0 + 2.1\left(\frac{f_{\ell e}}{f'_{co}}\right)^{0.7}$$

$$\varepsilon_{cc} = \varepsilon_{co} + 0.21\left(\frac{f_{\ell e}}{f'_{co}}\right)^{1.7}$$

$$\varepsilon_{cc50} = \varepsilon_{o50} + 0.15\left(\frac{f_{\ell e}}{f'_{co}}\right)^{1.1}$$

$$f_{hcc} = f_{yh}$$

$$\varepsilon_{o50} = 0.004$$

$$f_{\ell e} = K_e f_\ell = \frac{K_e f_{hcc}}{s}\left(\frac{A_{shx} + A_{shy}}{b_{cx} + b_{cy}}\right)$$

$$K_e = \frac{\left[1 - \sum_{i=1}^{n} \frac{(w_i)^2}{6 b_{cx} b_{cy}}\right]\left(1 - 0.5\frac{s'}{b_{cx}}\right)\left(1 - 0.5\frac{s'}{b_{cy}}\right)}{1 - \rho_t}$$

Efective confinement index = $IP_e = f_{\ell e}/f'_{co}$

$$\varepsilon_{hcc} = 0.5\varepsilon_{cc}\left[1 - (f_{\ell e}/f'_{cc})\right]$$

in which, A_{shx} is the area of lateral reinforcement at cross section perpendicular to *x*-axis, A_{shy} the area of lateral reinforcement at cross section perpendicular to *y*-axis, f_ℓ the stress of nominal lateral reinforcement acting in concrete core, $f_{\ell e}$ the effective confining stress acting in concrete core, f_{hcc} the stress of lateral reinforcement at peak stress of confined concrete, k the coefficient affecting the slope of ascending branch of stress-strain curve, k_1 the coefficient affecting the slope of descending branch of stress-strain curve, k_2 the coeffient affecting the curvature of descending branch of stress-strain curve, and ε_{hcc} the strain of lateral reinforcement when the stress reaches f_{hcc}. All units are in SI system.

Diniz-Frangopol Model [12]

For ascending branch, $\varepsilon_c \le \varepsilon_{cc}$:

$$f_c = f'_{cc}\left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{cc}}\right)^A\right] \quad (18)$$

For descending branch, $\varepsilon_c \ge \varepsilon_{cc}$:

$$f_c = f'_{cc}\exp\left[-k(\varepsilon_c - \varepsilon_{cc})^{1.15}\right] \quad (19)$$

where:

$$f_\ell = \frac{A_{sh} f_{yh}}{d_e s}$$

$$A_{sh} = \lambda A_{st}$$

$$f_{\ell e} = C_f f_\ell$$

$$C_f = 1 - \frac{s}{d_e}$$

$$A = E_c.\varepsilon_{cc}/f'_{cc}$$

$$E_c = 33 w_c^{1.5}\sqrt{f'_c}$$

$$k = 0.17 f'_c \exp(-0.01 f_{\ell e}/\lambda_1)$$

$$\lambda_1 = 1 + 25\frac{f_{\ell e}}{f'_c}\left[1 - \exp\left(f'_c/44.79\right)^9\right]$$

$$f'_{cc} = f'_c + \left(1.15 + \frac{21}{f'_c}\right) f_{\ell e}$$

$$\varepsilon_{cc} = 1.027\times10^{-7} f'_c + 0.0296\frac{f_{\ell e}}{f'_c} + 0.00195$$

in which, d_e is the equivalent diameter of lateral reinforcement, A_{sh} the total cross-sectional area of lateral reinforcement in a section including crossties, A_{st} the cross-sectional area of lateral reinforcement, C_f the corrective factor for confinement, and λ a factor depending on the configuration type of lateral reinforcement. All units are in SI system.

Kappos-Konstantinidis Model [13]

For ascending branch, $0 < \varepsilon_c \le \varepsilon_{cc}$:

$$f_c = \frac{f'_{cc}\left(\frac{\varepsilon_c}{\varepsilon_{cc}}\right)\left(\frac{E_c}{E_c - E_p}\right)}{\left(\frac{E_c}{E_c - E_p}\right) - 1 + \left(\frac{\varepsilon_c}{\varepsilon_{cc}}\right)^{\frac{E_c}{E_c - E_p}}} \quad (20)$$

For descending branch, $\varepsilon_c > \varepsilon_{cc}$:

$$f_c = f'_{cc}\left[1 - 0.5\frac{\varepsilon_c - \varepsilon_{cc}}{\varepsilon_{cc50} - \varepsilon_{cc}}\right] \ge 0.3 f'_{cc} \quad (21)$$

$$E_c = 22{,}000\left(\frac{f'_c}{10}\right)^{0.3} \text{ (in MPa)}$$

$$E_p = \frac{f'_{cc}}{\varepsilon_{cc}}$$

$$f'_{cc} = f'_{co} + 10.3(\alpha\rho_s f_{yh})^{0.4}$$

$$f'_{co} = 0.85 f'_c$$

$$\varepsilon_{cc} = \left[1 + 32.83(\alpha\omega_w)^{1.9}\right]\varepsilon_{co}$$

$$\varepsilon_{co} = \frac{0.70(f'_c)^{0.31}}{1{,}000}$$

$$\omega_w = \frac{\rho_s f_{yh}}{f'_c}$$

$$\alpha = \left(1 - \frac{\sum (b_i)^2}{6 b_c d_c}\right)\left(1 - \frac{s}{2b_c}\right)\left(1 - \frac{s}{2d_c}\right)$$

$$\varepsilon_{cc50} = \varepsilon_{co} + 0.0911(\alpha\omega_w)^{0.8}$$

in which, α is a factor accounting for the effectiveness of confinement, ω_w the mechanical volumetric ratio of lateral reinforcement, b_i the distance between two adjacent longitudinal reinforcement measured center-to-center of reinforcement, $\alpha\omega_w$ the effective capacity of lateral reinforcement, and E_p the secant modulus of elasticity of concrete at peak stress.

Hong-Han Model [14]

For ascending branch, $0 < \varepsilon_c \le \varepsilon_{cc}$:

$$f_c = f'_{cc}\left\{1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{cc}}\right)^{\alpha}\right\} \quad (22)$$

For descending branch, $\varepsilon_c > \varepsilon_{cc}$:

$$f_c = f'_{cc} - E_{des}(\varepsilon_c - \varepsilon_{cc}) \quad (23)$$

where:

$$E_{des} = 0.026\frac{f'^{3}_{co}}{f_{\ell e}^{0.4}}$$

$$\alpha = E_c\frac{\varepsilon_{cc}}{f'_{cc}}$$

$$E_c = 3320\sqrt{f'_{co}} + 6900$$

$$\frac{f'_{cc}}{f'_{co}} = 1.0 + 4.1\left(\frac{f_{\ell e}}{f'_{co}}\right)^{0.70}$$

$$\varepsilon_{cc} = \varepsilon_{co} + 0.015\left(\frac{f_{\ell e}}{f'_{co}}\right)^{0.56}$$

$$f_{\ell e} = K_e\rho_s f_{hcc}$$

$$K_e = \frac{\left[1 - \sum_{i=1}^{n}\frac{(w_i)^2}{6b_{cx}b_{cy}}\right]\left(1 - 0.5\frac{s'}{b_{cx}}\right)\left(1 - 0.5\frac{s'}{b_{cy}}\right)}{1 - \rho_t}$$

$$f_{hcc} = E_s\left\{0.45\varepsilon_{co} + 0.73\left(\frac{K_e\rho_s}{f'_{co}}\right)^{0.70}\right\} \le f_{yh}$$

$$f'_{co} = 0.85 f'_c$$

$$\varepsilon_{co} = 0.0028 - 0.0008 k_3$$

$$k_3 = 40/f'_{co} \le 1.0$$

in which, E_s is the modulus of elasticity of lateral reinforcement. All units are in SI system.

Kusuma-Tavio Model [15]

For ascending branch, $\varepsilon_c \le \varepsilon_{cc}$:

$$f_c = f'_{cc}\frac{K_b\varepsilon_b - \varepsilon_b^2}{1 + (K_b - 2)\varepsilon_b} \quad (24)$$

For descending branch, $\varepsilon_c > \varepsilon_{cc}$:

$$f_c = f'_{cc} - E_{des}(\varepsilon_c - \varepsilon_{cc}) \quad (25)$$

where:

$$K_b = \frac{E_c\varepsilon_{cc}}{f'_{cc}}$$

$$\varepsilon_b = \frac{\varepsilon_c}{\varepsilon_{cc}}$$

$$E_c = 0.043 w_c^{1.5}\sqrt{f'_c} \text{ (in MPa)}$$

$$f_{\ell e} = 0.5 k_e\,\rho_s\,f_{yh}$$

$$k_e = \left(1 - \frac{\sum b_i^2}{6 b_c\, d_c}\right)\left(1 - \frac{s}{b_c}\right)^2$$

$$E_{des} = \frac{12.2}{\rho_s f_{yh}/(f'_c)^2}$$

$$\varepsilon_{cu} = \varepsilon_{cc} + \frac{f'_{cc}}{2E_{des}}$$

in which, E_{des} is the strength reduction factor, k_e a factor accounting for effectiveness of confinement, and s the spacing of lateral reinforcement measured center-to-center of reinforcement.

Effects of Confinement on Stress-Strain Curves of Concrete

The effects of confinement on stress-strain curves of concrete are investigated using a program developed by the authors, namely ConfinedCOL v.1 [19], and the results are shown in Fig. 1. The presence of closely-spaced lateral confinement significantly increases the magnitude of stress-strain curve of concrete. Summary of the effects of confinement parameters on the stress-strain curves of concrete according to several models proposed earlier are given in Table 1. The most influencing parameter is found to be the spacing of transverse steel.

Effects of Confinement on Column Capacity

The effects of confinement directly influence the shape and magnitude of stress-strain curve of concrete. This in turn will affect the compresive force per unit width of concrete, c_c. This gain further increases the compressive force of concrete, C_c, as follows:

$$C_c = c_c b \tag{26}$$

where c_c is the compressive force of concrete per unit width (N/mm), and b the width of compressive section (mm). The increase of the compressive force of concrete (C_c) will automatically improve the nominal capacity of a column subjected to axial load (P) and bending moment (M), or in other words, the interaction diagram of the column is enlarged.

The effects of confinement on the strength gain due to the presence of confinement through the requirement of minimum lateral reinforcement have already been considered in the building code. However, this strength gain is used only for the compensation for the possible strength loss due to the spalling of concrete cover (which is unconfined). Recent codes of practice still disregard this effect for the design purposes and, thus in the conservative side. In the proposed analytical models, both the strength gain in concrete core and the loss of strength in the cover are considered to exhibit the remaining strength gain after the mobilization of strength gain in the core concrete to compensate the loss of strength in the concrete cover. For confined concrete columns, the behavior of concrete core is modeled by the stress-strain relationship of confined concrete, whereas for the cover, it is assumed as unconfined concrete.

Effects of Confinement on Interaction Diagram of Concrete Columns

To investigate the amount of capacity gain in axial load and bending moment due to the confinement effects, an analytical study is conducted on a column model with the following data: (a) unconfined concrete compressive strength, f_c': 30 and 60 MPa, (b) cross section: width (B) and depth (H), 400 mm, (c) longitudinal reinforcement: 8, 19 mm diameter bars (ρ_t = 1.43 percent), (d) lateral reinforcement: diameter 10 mm, (e) concrete cover 40 mm, (f) spacing of lateral reinforcement 100 mm, (g) yield strength of lateral reinforcement, f_{yh}: 240 MPa, and (h) yield strength of longitudinal reinforcement, f_{yl}: 240 MPa. The interaction diagram is also constructed using ConfinedCOL v.1 [19]. All models discussed in the foregoing section are used to observe the effects of confinement of each model on the capacity gain of a column. From the results of the analysis, it indicates that there is a capacity gain in axial load and bending moment of confined concrete column compared to that of the unconfined one, particularly in the compression-controlled region shown in Fig. 2. For instance, the capacity gain is shown in hatched region in Fig. 3 according to the latest model [14]. This is due to the expansion of the area of compressive concrete in a column section with the presence of confinement produced by lateral reinforcement. Note that for confined concrete columns, the behavior of concrete core is modeled by the stress-strain relationship of confined concrete, whereas for the cover, it is assumed as unconfined concrete. The relevant stress-strain models are adopted in the analysis to properly accommodate both regions of concrete cross-section.

Recent building code requires closely-spaced lateral reinforcement in a reinforced concrete column to satisfy the ductility and strength requirements of a seismic-resistant building [20]. Even though the codes ignore the effect of confinement on the strength gain due to the conservative consideration for the design purposes, with the capacity gain due to confinement effects shown in the analysis, the authors still expect that a reinforced concrete column could resist higher axial load and bending moment in the future design. Tables 2 and 3 show the substantial capacity gains of confined concrete columns compared to the unconfined one in terms of axial load and bending moment using the adopted models after the mobilization of strength gain in the core concrete to compensate the loss of strength in the concrete cover. The use of several stress-strain model of unconfined concrete does not demonstrate significant difference in terms of strength at this state.

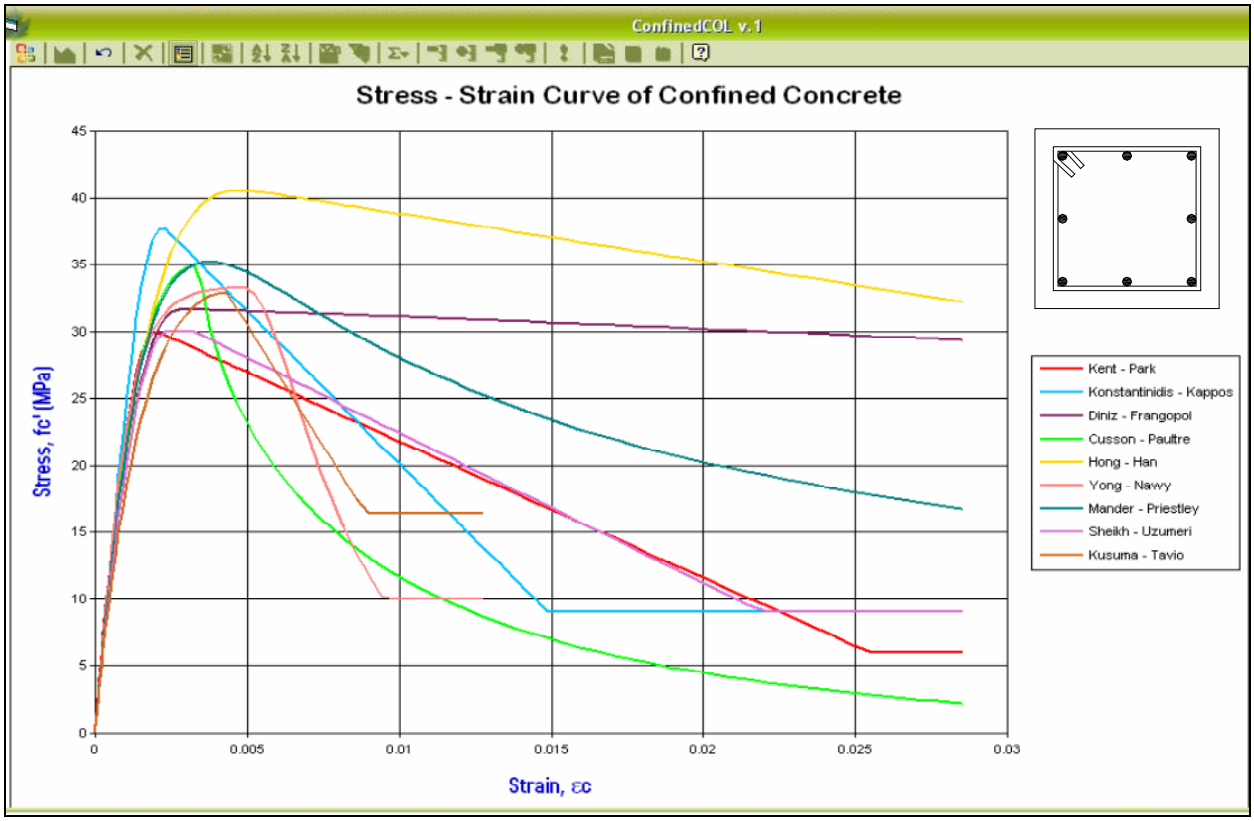
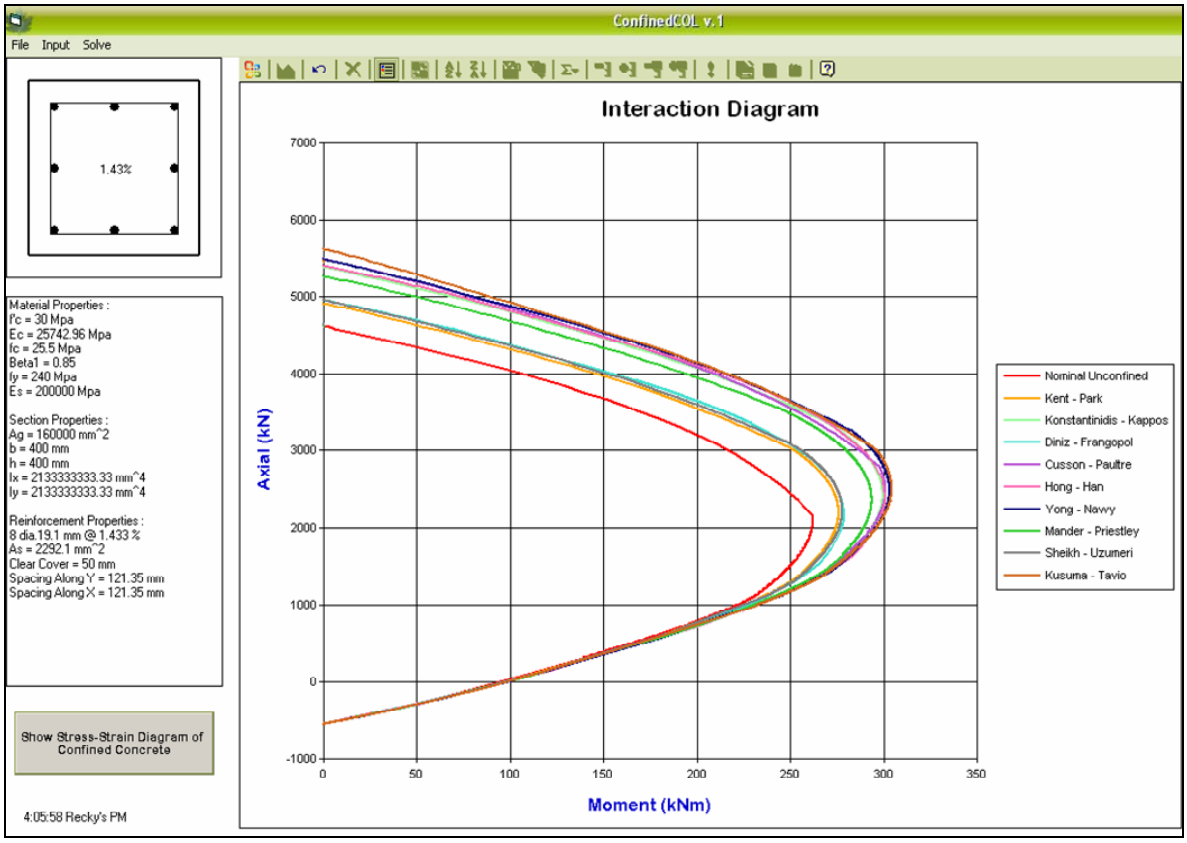


Figure 1. Effects of confinement on stress-strain curves of concrete

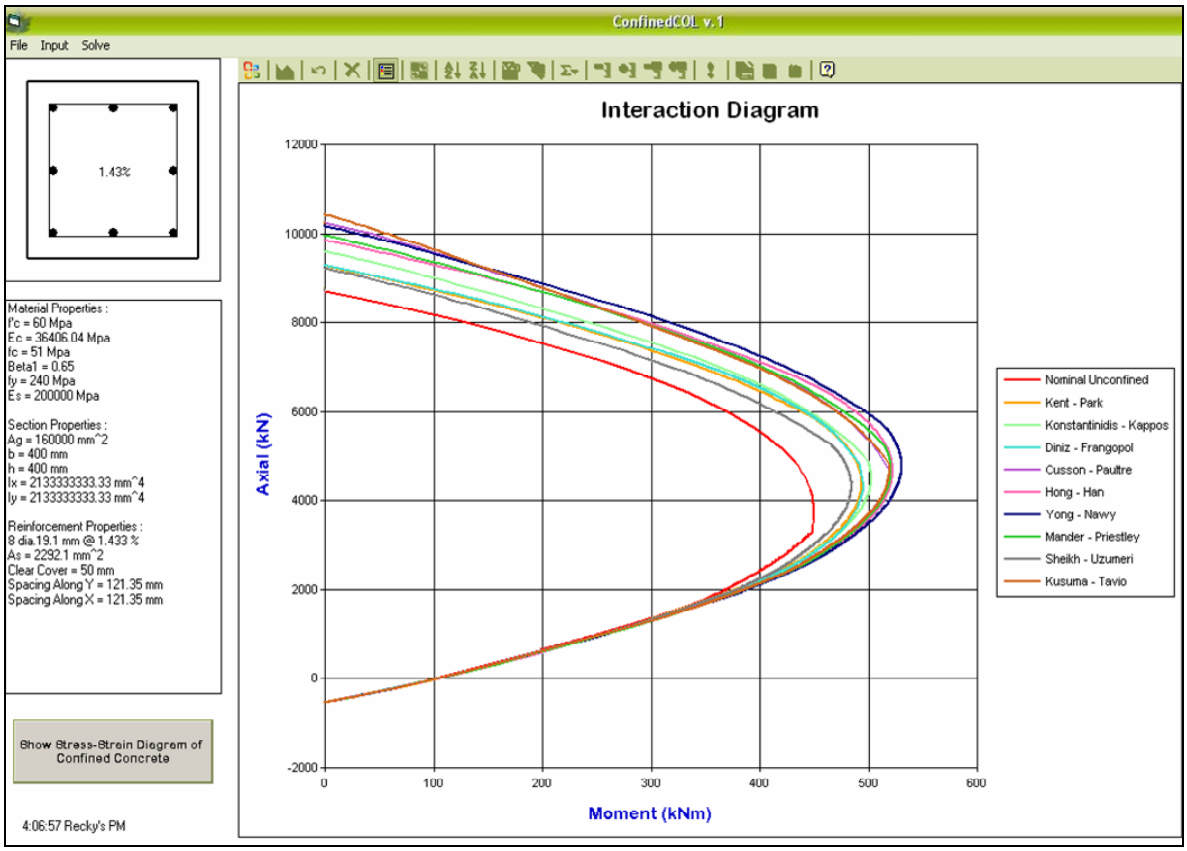
Table 1. Summary of effects of confinement parameters on stress-strain curves of concrete

Confinement Model	Parameters	Confinement Parameters					
		Diameter of Lateral Reinforcement	Spacing of Lateral Reinforcement	Yield Strength of Lateral Reinforcement	Configuration of Lateral Reinforcement	Number of Longitudinal Reinforcement	Configuration of Longitudinal Reinforcement
Kent-Park	Peak Stress	–	–	–	–	–	–
	Peak Strain	–	–	–	–	–	–
	Ultimate Strain	+	+	–	+	–	–
Sheikh-Uzumeri	Peak Stress	+	+	+	+	+	+
	Peak Strain	+	+	+	+	+	+
	Ultimate Strain	+	+	+	+	+	+
Mander et al.	Peak Stress	+	+	+	+	+	+
	Peak Strain	+	+	+	+	+	+
	Ultimate Strain	+	+	+	+	+	+
Yong et al	Peak Stress	+	+	+	+	+	–
	Peak Strain	+	+	+	+	–	–
	Ultimate Strain	+	+	+	+	+	–
Cusson-Paultre	Peak Stress	+	+	+	+	+	+
	Peak Strain	+	+	+	+	+	+
	Ultimate Strain	+	+	+	+	+	+
Diniz-Frangopol	Peak Stress	+	+	+	+	–	–
	Peak Strain	+	+	+	+	–	–
	Ultimate Strain	+	+	+	+	–	–
Kappos-Konstantinidis	Peak Stress	+	+	+	+	+	+
	Peak Strain	+	+	+	+	+	+
	Ultimate Strain	+	+	+	+	+	+
Hong-Han	Peak Stress	+	+	+	+	+	+
	Peak Strain	+	+	+	+	+	+
	Ultimate Strain	+	+	+	+	+	+
Kusuma-Tavio	Peak Stress	+	+	+	+	+	+
	Peak Strain	+	+	+	+	+	+
	Ultimate Strain	+	+	+	+	+	+

Notes: + = affecting, — = not affecting.



(a) f_c' **= 30 MPa**



(b) f_c' **= 60 MPa**

Figure 2. Comparisons between unconfined and confined interaction diagrams of concrete columns

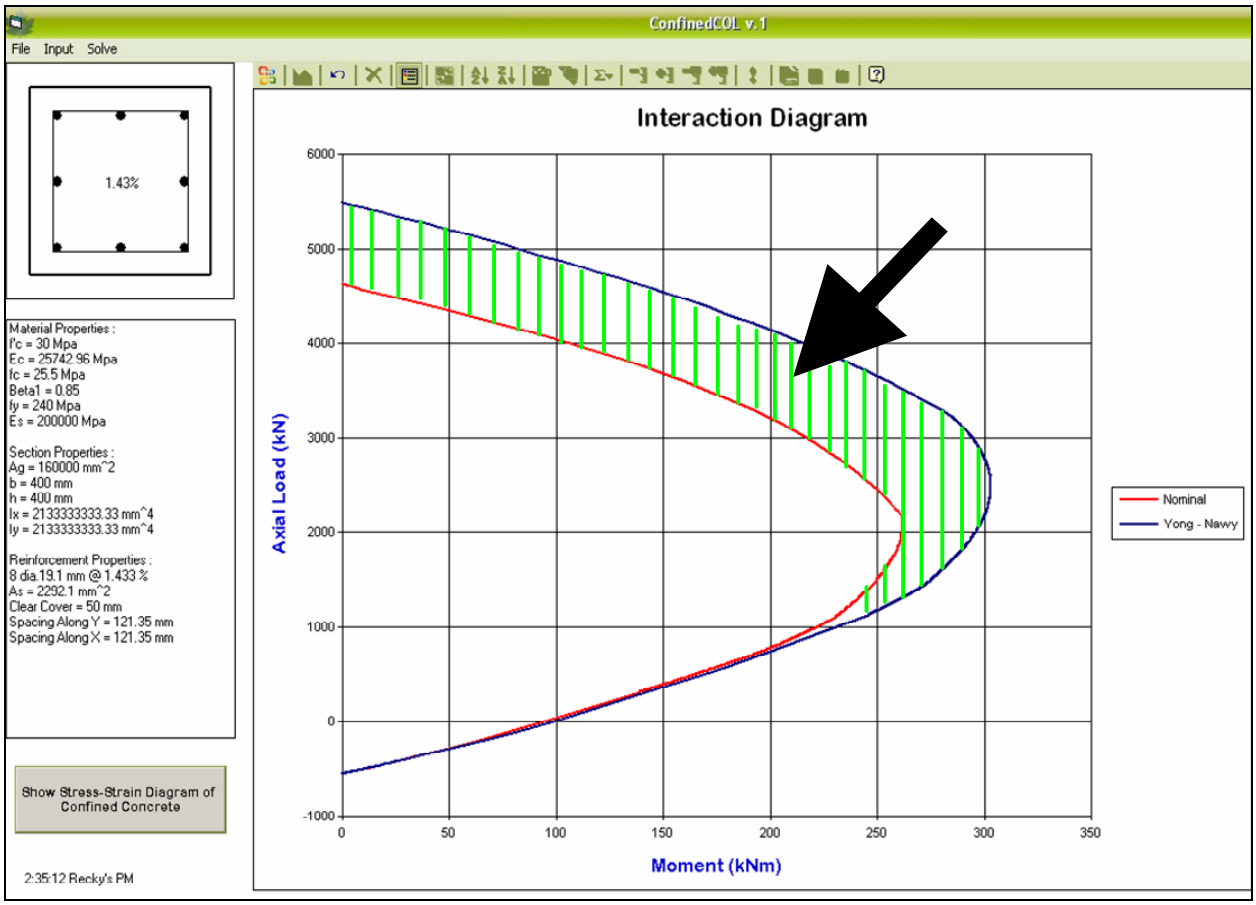


Figure 3. Capacity gain of confined concrete columns in compression-controlled region

Conclusions

From the study, the following conclusions can be drawn:

1. Three key parameters affecting the shape and magnitude of stress-strain curve of concrete are the peak stress, the strain at peak stress, and the ultimate strain.
2. It can be concluded that there are six key parameters primarily influence the effectiveness of lateral confinement. The most influencing parameter is found to be the spacing of transverse steel.
3. There is still a possible remaining capacity gain in axial load and bending moment of confined concrete column compared to that of the unconfined one, particularly in the compression-controlled region, after the mobilization of strength gain in the core concrete to compensate the loss of strength in the concrete cover.
4. Even though, the codes ignore the effect of confinement on the strength gain due to the conservative consideration for the design purposes, with the remaining capacity gain found due to confinement effects, the authors still hope that in the future design a more economical reinforced concrete column can be expected to resist higher axial load and bending moment by maintaining its size without any enlargement, particularly for lower-story columns which are dominated by the axial load rather than flexure.
5. Further study needs to be carried out in the future, particularly in three dimensional models to capture the three dimensional cracking/ fracturing behavior of concrete to confirm that the capacity gain of a column could be accounted for in the future design codes with confidence.

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References

1. Villaverde, R., Methods to Assess the Seismic Collapse Capacity of Building Structures: State of the Art, *Journal of Structural Engineering*, ASCE, V.133, No. 1, January 2007, pp. 57-66.

2. Whitney, C. S., Design of Reinforced Concrete Members under Flexure or Combined Flexure and Direct Compression, *ACI Journal*, March 1937, V. 33, No. 3, pp. 483-498.

3. ACI Committee 318, *Building Code Requirements for Structural Concrete (ACI 318-56),* American Concrete Institute, Detroit, Michigan, 1956.

4. Departemen PU, *Tata Cara Perhitungan Struktur Beton untuk Bangunan Gedung (SK SNI T-15-1991-03)*, Yayasan LPMB, Bandung, 1991.

5. Purwono, R., Tavio; Imran, I., and Raka, I G. P., *Tata Cara Perhitungan Struktur Beton untuk Bangunan Gedung (SNI 03-2847-2002) Dilengkapi Penjelasan (S-2002)*, ITS Press, Surabaya, March 2007.

6. Fanella, D. A., Munshi, J. A., and Rabbat, B. G., *Notes on ACI 318-99 Building Code Requirements for Structural Concrete with Design Applications,* Portland Cement Association, Skokie, Illinois, 1999.

7. Kent, D. C., and Park, R., Flexural Members with Confined Concrete, *Journal of Structural Division*, ASCE, V. 97, No. ST7, July 1971, pp. 1969-1990.

8. Sheikh, S. A., and Uzumeri, S. M., Analytical Model for Concrete Confinement in Tied Columns, *Journal of the Structural Division*, ASCE, V. 108, No. ST12, Dec. 1982, pp. 2703-2722.

9. Mander, J. B., Priestley, M. J. N., and Park, R., Theoretical Stress-Strain Model for Confined Concrete, *Journal of the Structural Division*, ASCE, V. 114, No. ST8, Aug. 1988, pp. 1804-1825.

10. Yong, Y. K., Nour, M. G., and Nawy, E. G., Behavior of Laterally Confined High-Strength Concrete under Axial Loads, *Journal of the Structural Division*, ASCE, V. 114, No. ST2, February 1988, pp. 332-351.

11. Cusson, D., and Paultre, P., Stress-Strain Model for Confined High-Strength Concrete, *Journal of Structural Engineering*, ASCE, V. 121, No. 3, March 1995, pp. 468-477.

12. Diniz, S. M. C., and Frangopol, D. M., Strength and Ductility Simulation of High-Strength Concrete Columns, *Journal of Structural Engineering*, ASCE, V. 123, No. 10, October 1997, pp. 1365-1374.

13. Kappos, A. J., and Konstantinidis, D., Statistical Analysis of Confined High-Strength Concrete Columns, *Material and Structures*, V. 32, Dec. 1992, pp. 734-748.

14. Hong, K. N., and Han, S. H., Stress-Strain Model of High-Strength Concrete Confined by Rectangular Ties, *Journal of Structural Engineering*, KSCE, V. 9, No. 3, 2005, pp. 225-232.

15. Kusuma, B., and Tavio, Unified Stress-Strain Model for Confined Columns of Any Concrete and Steel Strengths, *Proceeding of the International Conference on Earthquake Engineering and Disaster Mitigation*, 14-15 Apr. 2008, Jakarta, Indonesia, pp. 502-509.

16. Popovics, S., A Numerical Approach to the Complete Stress-Strain Curve for Concrete, *Cement and Concrete Research*, V. 3, No. 5, 1973, pp. 583-599.

17. Thorensfeldt, E., Tomaszewicz, A., and Jensen, J. J., Mechanical Properties of High-Strength Concrete and Application in Design, *Proceedings of the Symposium Utilization of High Strength Concrete*, Tapir, Trondheim, 1987, pp. 149-159.

18. Tavio, Budiantara, I N., and Kusuma, B., Spline Nonparametric Regression Analysis of Stress-Strain Curve of Confined Concrete, *Civil Engineering Dimension, Journal of Civil Engineering Science and Application*, V. 10, No. 1, Department of Civil Engineering, Petra Christian University, Surabaya, March 2008, pp. 14-27.

19. Tirtajaya, R., *Analisis Penampang Kolom Beton Bertulang Menggunakan Visual Basic 6.0 dengan Memperhitungkan Efek Pengekangan*, Final Project, Department of Civil Engineering, Sepuluh Nopember Institute of Technology (ITS), Surabaya, August 2008.

20. Purwono, R., and Tavio, *Evaluasi Cepat Sistem Rangka Pemikul Momen Tahan Gempa*, ITS Press, Surabaya, September 2007.