Comparative Performance Analysis of Linear Precoding in Downlink Multi-user MIMO

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***Abstract*—This paper investigates the comparative performance of linear precoding schemes. The linear precoding schemes are including block diagonalization (BD), zero forcing (ZF), and maximum ratio transmission (MRT) in downlink multi-user MIMO. This work delivers the performance of linear precoding in term of achievable sum rate and bit error rate (BER) with a variation of the signal to noise ratio (SNR) and the number of transmitter-receiver antennas. We suppose that the transmitters have a complete channel state information. The results show that the MRT precoding yields better bit error rate than both the BD and ZF precoding schemes. However, the ZF precoding generates better achievable sum rate than the MRT precoding. In the other side, the MRT precoding also outperforms when the number of active users is bigger than K_{cross} while the number of active users is less than K_{cross} the ZF precoding is still dominant.**

Keywords—precoding, maximum ratio transmission, zero forcing, block diagonalization, BER, sum rate

I. INTRODUCTION

Recently, the number of users on mobile communication system is exponentially increasing. Users need a higher data rate (gigabit per second), low latency and full mobility communication services. Mobile communication technology has to transform their infrastructure to accommodate the demands. Mobile communication system is now moving into 5^{th} generation. The 5^{th} generation is operating on the millimeter wave spectrum to reach a wider bandwidth but there are some wave propagation challenges. In addition, the 5^{th} generation also deploys the newest subsystems on its infrastructure, one of them is an antenna subsystem. The newest mobile communication technology is implementing a multiple-input multiple-output (MIMO) antenna subsystem. The MIMO antenna is transformed into a massive MIMO antenna when a large number of the antenna in one transmitter. The massive MIMO have some advantages in term of channel capacity, spectral efficiency, interference minimization and link reliability [1]. The massive MIMO has been a hot issue of research due to its capability to increase capacity, spectral, reliability and minimize the interference. The transmitter is equipped with large number antennas (massive MIMO) serves a several single or multiple receiver antennas users, it is called a multi-user MIMO. Several users are served simultaneously, there is multipath between transmitter and receiver. Interference could potentially appear under these conditions. The advancement of massive MIMO could be realized by adding a precoding/beamforming subsystem. The precoding plays a key role in multi-user MIMO signal processing. The precoding consists of two types which are non-linear and linear. Many previous studies have reported the performance of both non-linear and linear precoding in multi-user MIMO. The authors in [2] delivered the performance of zero forcing precoding. In [3], the authors analyzed the comparison between vector normalization and matrix normalization for maximum ratio transmission precoding. Meanwhile, the author in [4] investigated the performance of minimum mean-square error (MMSE) detector and zero forcing in term of spectral efficiency in downlink massive MIMO. In [5], the authors discussed the performance of statistical and imperfect channel state information (CSI) combination for non-linear precoding in downlink massive MIMO. In [6], the authors optimized the average minimum mean square error (AMMSE) detector with imperfect CSI in multi-user MISO (multiple-input single-output).

This paper investigates the linear precoding with complete channel state information-transmitter (CSIT) in downlink multi-user MIMO. Linear precoding is including block diagonalization (BD), maximum ratio transmission (MRT) and zero-forcing (ZF).

This paper is organized as follows. The introduction, previous works, and background of this research are delivered in section I. The multi-user MIMO system model of this work is detailed in section II. The results and discussion of linear precoding performance will be presented in Section III. Section IV will conclude this work.

II. SYSTEM MODEL

This work uses a single cell model. Fig 1 depicts the single cell model. The transmitter is equipped with multiple antennas. The transmitter has perfect CSI for all users. The channel of multi-user MIMO uses Rayleigh channel model. The channel that coupling the transmitter and the users is depicted in Fig. 2. In addition, the precoding position is also shown in Fig. 2. The system model of this work in detail is described in Fig. 3.

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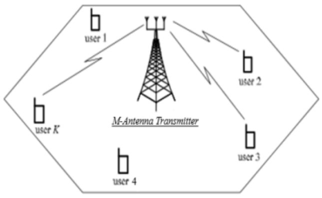

Fig. 1. Multi-user MIMO transceiver (single cell)

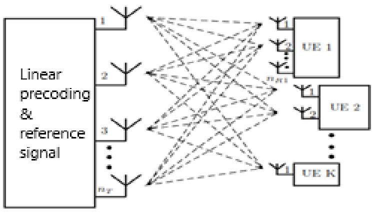

Fig. 2. Precoding subsystem

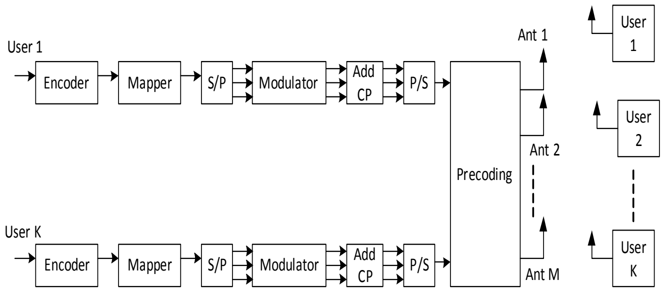

Fig. 3. Multi-user MIMO system model

Consider the downlink multi-user MIMO channels with K-single antennas users are served by a transmitter that equipped with M antennas. The transmitter simultaneously serves K number of J_k antennas users. The channel matrix between the k^{th} user to the transmitter is modeled as follows [3][7]:

$$H_k = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & \cdots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{J_k 1} & h_{J_k 2} & \cdots & h_{J_k M} \end{bmatrix} \quad (1)$$

$$H = [H_1^T \; H_2^T \; H_3^T \; \cdots \; H_{k-1}^T \; H_k^T]^T \quad (2)$$

where, $H_k \in C^{J_k \, x \, M}$ is multi-user MIMO channel matrix response between the transmitter and k^{th} user. H is the multi-user MIMO system channel matrix. Y_k is the received signal of k^{th} user which is formulated by [7]:

$$Y_k = H_k P_k S_k + H_k \sum_{a=1, a \neq k}^{K} P_a S_a + n_k \quad (3)$$

S_k is the k^{th} user transmission symbol vector with a set $\left[S_k^1 \; S_k^2 \; S_k^3 \; \cdots \; S_k^{r_k}\right]$. n_k is the k^{th} user additive white Gaussian noise (AWGN) with σ^2 variance and zero-mean. P_k is the precoding matrix for k^{th} user. $P = [P_1 \; P_2 \; P_3 \; \cdots \; P_{k-2} P_{k-1} \; P_k]$ is a set of precoding multi-user MIMO system. $S = [S_1^T \; S_2^T \; S_3^T \; \cdots \; S_{k-1}^T \; S_k^T]^T$ is a set of transmission symbol of system.

Block diagonalization consists of two single value decomposition (SVD) operations. The first SVD operation will eliminate the multi-user interference (MUI) from the other users. The second SVD operation is applied in parallel user's data stream to maximize the precoding gain.

$$\widetilde{H}_k = [H_1^T \; \cdots \; H_{k-1}^T \; H_{k+1}^T \; \cdots H_K^T]^T \quad (4)$$

$$H_i P_k = 0, i \neq k \quad (5)$$

SVD decomposition of $\widetilde{H}_k$ is described as follows:

$$\widetilde{H}_k = \widetilde{U}_k \widetilde{\Sigma}_k \left[\widetilde{V}_k^{(1)} \; \widetilde{V}_k^{(0)}\right]^H \quad (6)$$

Where $\widetilde{V}_k^{(0)}$ is a singular matrix with zero singular value and $\widetilde{V}_k^{(1)}$ is a singular matrix with non-zero singular value.

$$P = \left[\widetilde{V}_1^{(0)} \; \widetilde{V}_1^{(1)} \; \widetilde{V}_2^{(0)} \; \widetilde{V}_2^{(1)} \cdots \widetilde{V}_{K-1}^{(0)} \; \widetilde{V}_{K-1}^{(1)} \; \widetilde{V}_K^{(0)} \; \widetilde{V}_K^{(1)}\right] \quad (7)$$

$$y = HPS + n = \begin{pmatrix} H_1P_1 & H_1P_2 & \cdots & H_1P_K \\ H_2P_1 & H_2P_2 & \cdots & H_2P_K \\ \vdots & \vdots & \vdots & \vdots \\ H_KP_1 & H_KP_2 & \cdots & H_KP_K \end{pmatrix} S + n = \begin{pmatrix} H_1P_1 & 0 & \cdots & 0 \\ 0 & H_2P_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & H_KP_K \end{pmatrix} S + n \quad (8)$$

Maximum ratio transmission (MRT) and zero forcing (ZF) precoding have been often implemented for multi-user MIMO signal processing because of a good performance and implementation simplicity. The precoding weight of ZF and MRT can be formulated by as follows, respectively [7][8].

$$W_{ZF} = H^H \, (HH^H)^{-1} = \mathrm{f}_1 \; \mathrm{f}_2 \; \mathrm{f}_3 \cdots \mathrm{f}_\mathrm{K} \quad (9)$$

$$W_{MRT} = H^H = \mathrm{f}_1 \; \mathrm{f}_2 \; \mathrm{f}_3 \cdots \mathrm{f}_\mathrm{K} \quad (10)$$

with K (the number of active users) and a large number of N (the number of antennas at the transmitter), the signal to interference plus noise ratio of k^{th} user, ZF and MRT precoding, is formulated as follows, respectively[7]:

$$SINR_{ZF, \, k^{th} \, user} = \frac{P_d(N-K)}{K} \quad (11)$$

$$SINR_{MRT, \, k^{th} \, user} = \frac{P_d \, N}{K(P_d + 1)} \quad (12)$$

Every active user in downlink multi-user MIMO has an achievable sum rate that is described as follows:

$$R_k = Log_2(1 + SINR_k) \quad (13)$$

The achievable sum rate of K users is formulated as :

$$R_{sum, K \, users} = K * Log_2(1 + SINR_k) \quad (14)$$

Formula (14) was applied in zero forcing, maximum ratio transmission, and block diagonalization precoding, there were described as follows, respectively[7][8]:

$$R_{ZF} = K * Log_2(1 + SNIR_k^{ZF}) \quad (15)$$

$$R_{ZF} = K * Log_2\left(1 + \frac{P_d(N-K)}{K}\right) \quad (16)$$

$$R_{MRT} = K * Log_2(1 + SNIR_k^{MRT}) \quad (17)$$

$$R_{MRT} = K * Log_2\left(1 + \frac{P_d\ N}{K(P_d + 1)}\right) \quad (18)$$

$$R_{BD} = {}_{W_s, H_j P_{j=0, i\neq j}}^{max} log_2 \left| I + \frac{1}{\sigma_n^2} H_s P_s W_s^* P_s^* \right| \quad (19)$$

III. Results and Discussion

In section III, this work delivers the performance of linear precoding in term of bit error rate (BER) and sum rate with a variation of SNR, a number of transmitter antennas, and a number of users.

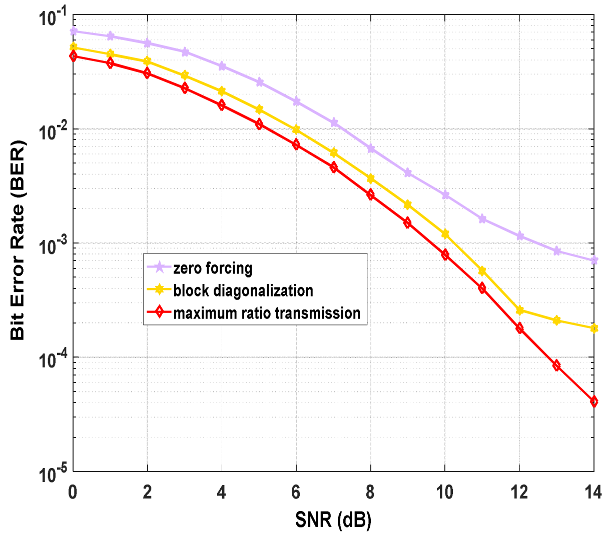

Fig. 4. BER performance with (2,2) x 4 antenna configuration

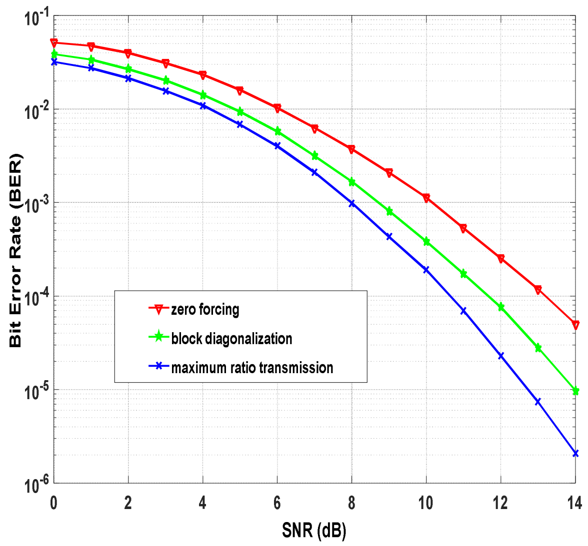

Fig. 5. BER performance with (3,3) x 6 antenna configuration

The results in Fig.4 and Fig.5 depict the comparison of BER performance of BD, ZF, and MRT precoding. The BER performance with variation of a transmit signal to noise ratio (SNR) is plotted. In Fig.4, we use the antenna configuration (2,2) x 4, while Fig.5 uses a (3,3) x 6 antenna configuration, respectively. We can see from Fig.4 and Fig.5, a bigger number of transceiver antennas produce better BER, at specific SNR 12 dB the (2,2) x 4 antenna configuration generates BER 1.34 10^{-4} (MRT) and the (3,3) x 6 antenna configuration yields BER 1.87 10^{-6} (MRT), respectively. There is about 10^{-2} of BER refinement. The MRT precoding also needs lower SNR than the ZF and BD precoding. For specific BER 10^{-4} at the (3,3) x 6 antenna configuration, MRT precoding needs about 10.3 dB of SNR while ZF and BD need about 13.5 dB and 11.6 dB. There are 3.2 dB and 1.3 dB of precoding gain. Furthermore, the MRT also has a lower computational complexity.

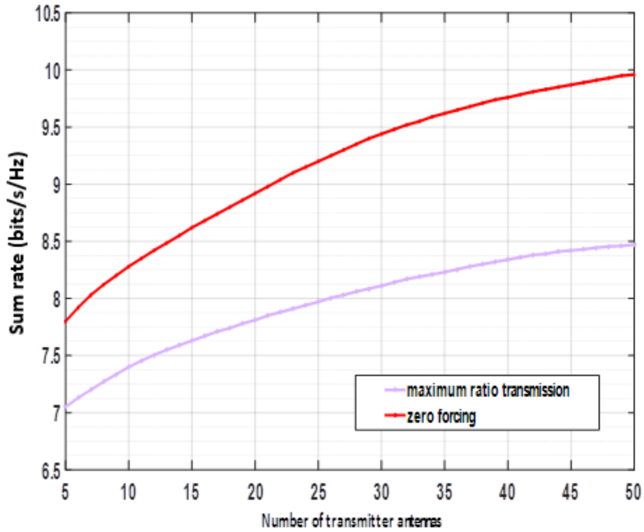

Fig. 6. Performance of achievable sum rate versus the number of transmitter antennas

Based on Fig.6, the results depict the performance of the achievable sum rate for both ZF and MRT precoding. Increasing the number of transmitter antennas lead the achievable sum rate will increase. In addition, a comparison of two precoding schemes indicates that ZF precoding yields higher sum rate than maximum ratio transmission precoding in multi-user MIMO system with equal power per user on downlink transmission.

In Fig 7, we illustrate the achievable sum rate as a function of the transmit SNR for both ZF and MRT precoding in downlink multi-user MIMO. We use the number active user (K) = 4 and the number of transmitter antennas (M) = 6. The MRT precoding outperforms at the low SNR, particularly 0 to 9.2 dB. The SNR 9.2 dB to be a turning point, ZF precoding gives better performance than MRT precoding at SNR 9.2 to 15 dB.

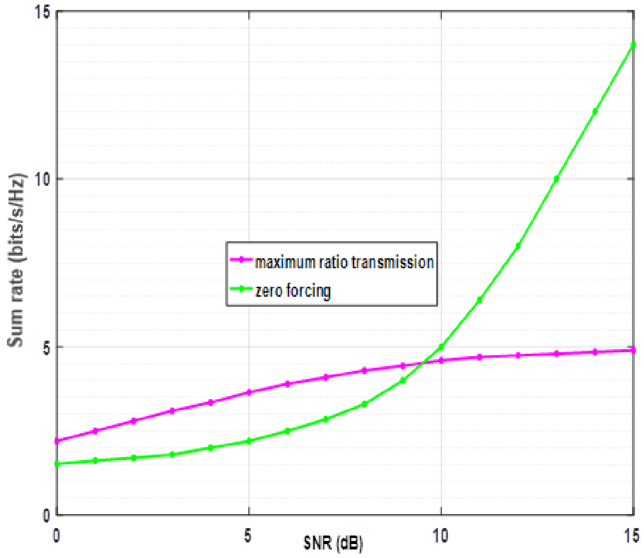

Fig. 7. Performance of achievable sum rate with a variation of SNR

Fig. 8 shows the comparison of the achievable sum rate as a function of a number of active users for both MRT and ZF precoding schemes with – 6 dB of power transmit SNR in downlink multi-user MIMO. As mentioned previously, in low SNR categories, the MRT precoding generates a better result with the number active users are larger than K_{cross} point, K_{cross} point was the number of active users that caused the curve of MRT and ZF were crossed. The MRT precoding performs increasingly as K increases. Whereas, the ZF precoding performance decreases as K increases.

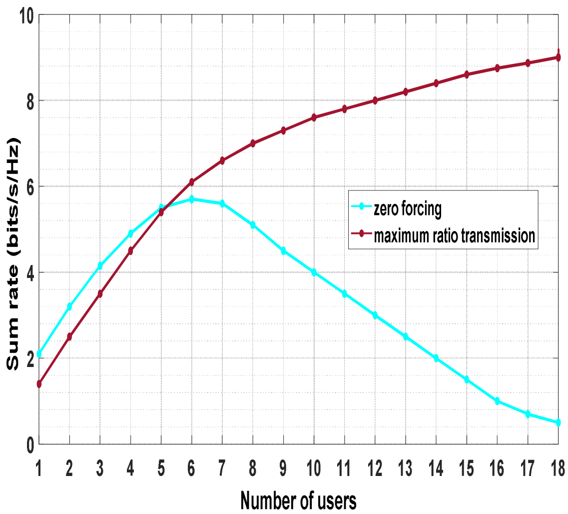

Fig. 8. Performance of achievable sum rate versus the number of active users

IV. Conclusions

This paper provides the comparison and analysis of linear precoding in single cell downlink multi-user MIMO. The investigated parameters are the bit error rate and the achievable sum rate with a variation in the number of active users and signal to noise ratio. Simulation results show that the MRT precoding scheme creates a better bit error rate. Meanwhile, The ZF precoding scheme gives better achievable sum rate. In addition, when a large number of active users is achieved (active users > K_{cross}) that the MRT precoding yields better achievable sum rate than the ZF precoding.

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